Using complete sentences and proper mathematical notation, state the **formal definition** of "critical number". **SCORE: _____ / 2 PTS**

C IS A CRITICAL NUMBER OF f IF AND ONLY IF C IS IN THE DOMAIN OF f AND $f'(c)=0$ OR IS UNDEFINED

Using complete sentences and proper mathematical notation, state the **formal definition** of "concave down". **SCORE: _____ / 2 PTS**

f IS CONCAVE DOWN ON $[a,b]$ IF AND ONLY IF f' IS DECREASING ON $[a,b]$

Find the global extrema of $f(x) = x^{\frac{2}{3}}(x-30)$ on the interval $[-1, 8]$. **SCORE: _____ / 6 PTS**

$f(x) = x^{\frac{5}{3}} - 30x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - 20x^{-\frac{1}{3}}$ IS UNDEFINED AT $x=0$ (1)

(-1) IF YOU FOUND $f(12)$

(1½) $= \frac{5}{3}x^{-\frac{1}{3}}(x-12) = 0$ AT $x = 12 \notin [-1,8]$ (1)

$f(-1) = (-1)^{2/3}(-1-30) = 1(-31) = -31$ (½)

$f(0) = 0^{2/3}(0-30) = 0(-30) = 0$ (½) ← MAX (½)

$f(8) = 8^{2/3}(8-30) = 4(-22) = -88$ (½) ← MIN (½)

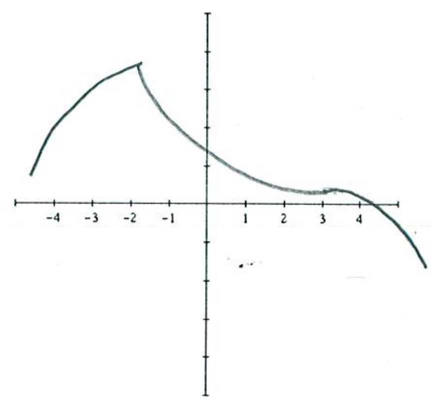
Sketch the graph of a **continuous** function $f(x)$ that satisfies all the following conditions. **SCORE: _____ / 4 PTS**

$f'(3) = 0$

$f'(x) < 0$ if $-2 < x < 3$ or $x > 3$, and $f'(x) > 0$ if $x < -2$

$f''(x) > 0$ if $-2 < x < 3$, and $f''(x) < 0$ if $x < -2$ or $x > 3$

f'	+		−	0	−
f''	−		+		−
		-2		3	



$f(x)$ is a polynomial function with derivative $f'(x) = (6+x)(8-x)^4$. **SCORE: _____ / 5 PTS**

[a] Find the critical numbers of f. **Justify your answer very briefly.**

$\frac{1}{2}$ $f'(x) = 0$ AT $x = -6, 8$ 1

[b] For each critical number of f, determine what the First Derivative Test tells you about that critical number. **Justify your answer very briefly.**

$1\frac{1}{2}$

		-6		8	
f'	$-$		$+$		$+$
$(6+x)$	$-$		$+$		$+$
$(8-x)^4$	$+$		$+$		$+$

$x = -6$ IS A LOCAL MIN 1

$x = 8$ IS NOT AN EXTREMA 1

$f(x)$ is a polynomial function such that $f'(-5) = f'(7) = 0$ and $f''(x) = (23-5x)(5+x)^3$. **SCORE: _____ / 3 PTS**

For each critical number of f, determine what the Second Derivative Test tells you about that critical number. **Justify your answer very briefly.**

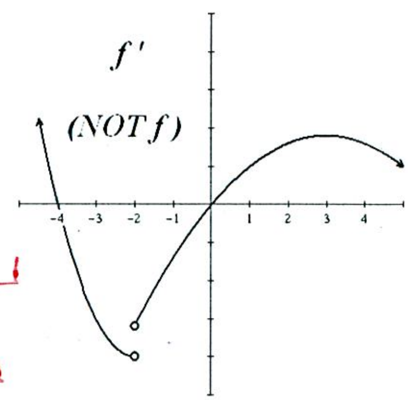
WRONG IF YOU SAID "NOT AN EXTREMA"

$f''(-5) = (23 - {-25})(5-5)^3 = 0$ $\frac{1}{2}$ $\to$ NO CONCLUSION 1

$f''(7) = (23-35)(5+7)^3 < 0$ $\frac{1}{2}$ $\to$ LOCAL MAX 1

$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right. **SCORE: _____ / 4 PTS**

The following questions are about the function f, **NOT THE FUNCTION f'**



[a] Write "I UNDERSTAND" if you understand that the following questions are about the continuous function f, **NOT THE FUNCTION f'**

[b] Find the critical numbers of f. **Justify your answer very briefly.**

1 $f'(x)$ DNE AT $x = -2$

1 $f'(x) = 0$ AT $x = -4, 0$

[c] Find the $x-$coordinates of all local minima of f. **Justify your answer very briefly.**

1 f' CHANGES FROM $-$ TO $+$ AT $x = 0$

[d] Find all intervals over which f is concave up. **Justify your answer very briefly.**

1 f' IS INCREASING ON $(-2, 3]$

Let $f(x)$ be a function such that $f(1) = 3$ and $f'(x) < 2$ for all $x \in [1, 5]$. **SCORE: _____ / 4 PTS**

Prove that $f(5) < 13$. **HINT: Write a proof by contradiction as shown in lecture.**

ASSUME $f(5) \geq 13$ 1

SINCE f' EXISTS ON $[1,5]$, f IS DIFFERENTIABLE + CONTINUOUS ON $[1,5]$ $\frac{1}{2}$

$\frac{1}{2}$ BY MVT, $f'(c) = \dfrac{f(5)-f(1)}{5-1} \geq \dfrac{13-3}{4} = 2\frac{1}{2}$ FOR SOME $c \in [1,5]$ 1

BUT $f'(x) < 2$ ON $[1,5]$, SO, BY CONTRADICTION, $f(5) < 13$. 1